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MHD Stagnation-Point Flow towards a Stretching Sheet with Prescribed Surface Heat Flux

(Aliran Titik Genangan MHD terhadap Helaian Meregang dengan Fluks Haba Permukaan Ditetapkan)

ANUAR ISHAK*, ROSLINDA NAZAR, NORIHAN M. ARIFIN, FADZILAH M. ALI & IOAN POP

ABSTRACT

The steady two-dimensional stagnation point flow of an incompressible viscous and electrically conducting fluid, subject to a transverse uniform magnetic field, towards a stretching sheet is investigated. The governing system of partial differential equations are transformed to ordinary differential equations, which are then solved numerically using a finite difference scheme known as the Keller-box method. The effects of the governing parameters on the flow field and heat transfer characteristics are obtained and discussed. It is found that the heat transfer rate at the surface increases with the magnetic parameter when the free stream velocity exceeds the stretching velocity, i.e. $\varepsilon>1$, *and the opposite is observed when* $\varepsilon<1$.

Keywords: Heat transfer; magnetohydrodynamic; stagnation point flow; stretching surface

ABSTRAK

Aliran titik genangan mantap dua matra bagi bendalir likat tak termampatkan dan pengkonduksi elektrik terhadap helaian meregang tertakluk kepada medan magnet melintang seragam diselidik. Sistem menakluk persamaan pembezaan separa dijelmakan kepada persamaan pembezaan biasa, yang kemudiannya diselesaikan secara berangka menggunakan skim beza terhingga dikenali sebagai kaedah kotak Keller. Kesan parameter-parameter menakluk terhadap medan aliran dan ciri-ciri pemindahan haba diperoleh dan dibincangkan. Didapati bahawa kadar pemindahan haba pada permukaan meningkat dengan peningkatan parameter magnet apabila halaju aliran bebas melebihi halaju regangan, iaitu $\varepsilon>1$, *dan keputusan yang bertentangan diperhatikan apabila* $\varepsilon<1$.

Kata kunci: Aliran titik genangan; magnetohidrodinamik; pemindahan haba; permukaan meregang

INTRODUCTION

The fluid dynamics due to a stretching surface is important in manufacturing processes. Examples are numerous and they include the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film in condensation processes, paper production, glass blowing, metal spinning and drawing plastic films. The quality of the final product depends on the rate of heat transfer at the stretching surface.

Since the pioneering study by Crane (1970) who presented an exact analytical solution for the steady two-dimensional stretching of a surface in a quiescent fluid, many authors have considered various aspects of this problem and obtained similarity solutions. Gupta and Gupta (1977) extended the problem posed by Crane (1970) to a permeable sheet and obtained closed-form solution, while Grubka and Bobba (1985) studied the thermal field and presented the solutions in terms of Kummer's functions. The 3-dimensional case has been considered by Wang (1984). Chen (1998) studied the case when buoyancy force is taken into consideration, and Magyari and Keller (1999) considered exponentially stretching surface. The heat transfer over a stretching surface with variable surface heat flux has been considered by Char and Chen (1988), Lin and Chen (1998), Elbashbeshy (1998), Ishak et al. (2008a), and very recently by Yacob and Ishak (2010). On the other hand, the flow over an unsteady stretching surface has been studied by Abbas et al. (2008) and Ishak et al. (2006, 2008b, 2009a, 2009b).

The steady laminar flow of an electrically conducting fluid caused by the stretching of an elastic sheet in the presence of a uniform magnetic field has been studied by Pavlov (1974). Andersson (1995) then demonstrated that the similarity solution derived by Pavlov (1974) is not only a solution of the boundary layer equations, but also represents an exact solution of the complete Navier-Stokes equations. Liu (2005) extended Andersson's results by finding the temperature distribution for non-isothermal stretching sheet, both in the prescribed surface temperature and prescribed surface heat flux conditions, in which the surface thermal conditions are linearly proportional to the distance from the origin. The flow and heat transfer characteristics over a stretching sheet in the presence of a uniform magnetic field has also been studied by Char

(1994), Chiam (1997), Liu (2004), Ishak et al. (2008c), and very recently by Prasad et al. (2009).

The above investigations considered the flow solely caused by a stretching sheet immersed in an otherwise quiescent fluid. In the present paper, we study the stagnation flow and heat transfer characteristics over a stretching sheet, with a uniform magnetic field is applied normal to it. The flow is not caused solely by the stretching sheet but also due to the external stream. The governing partial differential equations are transformed into ordinary differential equations using similarity transformation, and then solved numerically by a finite difference scheme, for some values of parameters. The results are then compared with those obtained by Elbashbeshy (1998) and Liu (2005) for some particular cases of the present study, to support their validity. It is worth mentioning that the MHD stagnation-point flow towards a stretching sheet but without considering the heat transfer aspect has been considered by Hayat et al. (2009, 2010).

MATHEMATICAL FORMULATION

Consider a steady, two-dimensional flow of an incompressible electrically conducting fluid near the stagnation point on a stretching sheet as shown in Figure 1. The stretching velocity $u_w(x)$ and the external velocity $u_e(x)$ are assumed to vary proportional to the distance x from the stagnation point O, i.e. $u_w(x) = ax$ and $u_e(x) = bx$, where a and b are constants with $a > 0$ and $b \geq 0$. It is also assumed that the surface of the sheet is subjected to a prescribed heat flux $q_w(x) = cx^n$, where c and n are constants with $c > 0$. Further, a uniform magnetic field of strength B_0 is assumed to be applied in the positive y-direction normal to the stretching sheet. The magnetic Reynolds number is assumed to be small, and thus the induced magnetic field is negligible. The simplified two-dimensional equations governing the boundary layer flow of a steady, laminar and incompressible viscous fluid are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho}\left(u_e - u\right), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where u and v are the velocity components along the x and y axes, respectively. Further, ν, ρ, α and T are respectively the kinematic viscosity, fluid density, thermal diffusivity and fluid temperature. We shall assume that the boundary conditions of Eqs. (1)-(3) are:

$$\begin{aligned} &u = u_w\left(x\right),\ v = 0,\ q_w\left(x\right) = -k\frac{\partial T}{\partial y} \text{ at } y = 0, \\ &u \to u_e\left(x\right),\ T \to T_\infty \text{ as } y \to \infty, \end{aligned} \tag{4}$$

where k is the thermal conductivity. The continuity equation (1) is satisfied by introducing a stream function ψ such that:

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}. \tag{5}$$

The momentum and energy equations can be transformed into the corresponding ordinary differential equations by the following transformation:

$$\eta = \left(\frac{a}{\nu}\right)^{1/2} y,\ f\left(\eta\right) = \frac{\psi}{\left(a\nu\right)^{1/2} x},\ \theta\left(\eta\right) = \frac{k\left(T - T_\infty\right)}{cx^n}\left(\frac{a}{\nu}\right)^{1/2}. \tag{6}$$

The transformed ordinary differential equations are:

$$f''' + ff'' - f'^2 + \varepsilon^2 + M(\varepsilon - f') = 0, \tag{7}$$

$$\frac{1}{Pr}\theta'' + f\theta' - nf'\theta = 0, \tag{8}$$

subject to the boundary conditions (4) which become

$$\begin{aligned} &f(0) = 0, f'(0) = 1, \theta'(0) = -1, \\ &f'(\eta) \to \varepsilon, \theta(\eta) \to 0 \text{ as } \eta \to \infty. \end{aligned} \tag{9}$$

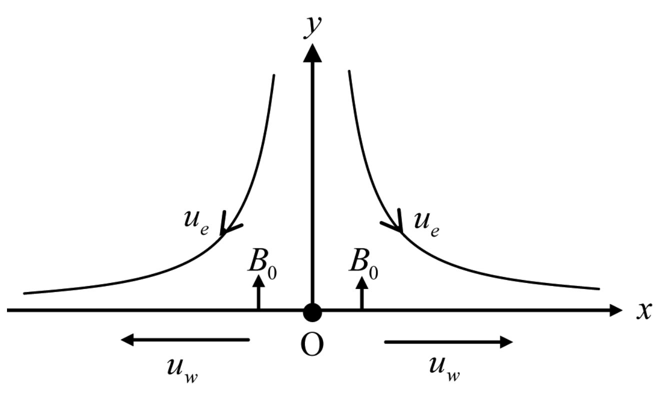

FIGURE 1. Physical model and coordinate system

Here primes denote differentiation with respect to η, $\varepsilon = b/a$ is the velocity ratio parameter, $Pr = \nu/\alpha$ is the Prandtl number and $M = \sigma B_0^2/(\rho a)$ is the magnetic parameter.

When $\varepsilon = 1$, the solution of (7) subject to the appropriate boundary conditions (9) is given by:

$$f(\eta) = \eta, \tag{10}$$

while when $\varepsilon = 0$, the solution was obtained by Andersson (1995) as:

$$f(\eta) = \frac{1}{\sqrt{1+M}}\left(1 - e^{-\sqrt{1+M}\eta}\right). \tag{11}$$

Further, for this case ($\varepsilon = 0$), the solution of Eq. (8) is given by:

$$\theta(\eta) = \frac{\sqrt{1+M}}{Pr} e^{-\frac{Pr}{\sqrt{1+M}}\eta} \frac{F\left(\frac{Pr}{1+M} - n, \frac{Pr}{1+M} + 1, -\frac{Pr}{1+M} e^{-\sqrt{1+M}\eta}\right)}{F\left(\frac{Pr}{1+M} - n, \frac{Pr}{1+M}, -\frac{Pr}{1+M}\right)}, \tag{12}$$

where $F(a, b, z)$ denotes the confluent hypergeometric function (see Abramowitz and Stegun (1965)), with:

$$F(a, b, z) = 1 + \sum_{n=1}^{\infty} \frac{a_n}{b_n} \frac{z^n}{n!},$$

$$a_n = a(a + 1)(a + 2) \ldots (a + n - 1),$$

$$b_n = b(b + 1)(b + 2) \ldots (b + n - 1).$$

Further, from Eqs. (11) and (12), the skin friction coefficient $f''(0)$ and the wall temperature $\theta(0)$ can be shown to be given by:

$$f''(0) = -\sqrt{1+M},$$

$$\theta(0) = \frac{\sqrt{1+M}}{Pr} \frac{F\left(\frac{Pr}{1+M} - n, \frac{Pr}{1+M} + 1, -\frac{Pr}{1+M}\right)}{F\left(\frac{Pr}{1+M} - n, \frac{Pr}{1+M}, -\frac{Pr}{1+M}\right)}. \tag{13}$$

We notice that when $\varepsilon = 0$ (quiescent fluid), the problem under consideration reduces to that considered by Char (1994) for the prescribed heat flux case. Moreover, when $\varepsilon = 0$ and $M = 0$ (without magnetic field) the present problem reduces to that of Chen and Char (1988) for impermeable stretching sheet, for which an exact analytical solution was reported, while when $\varepsilon = 0$, $M = 0$ and $n = 0$ (uniform surface heat flux), (7)-(9) reduce to that of Dutta et al. (1985), who investigated the heat transfer characteristic in an electrically non-conducting fluid past a stretching sheet with uniform surface heat flux.

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as:

$$C_f = \frac{\tau_w}{\rho u_w^2/2}, \quad Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, \tag{14}$$

where the wall shear stress τ_w and the surface heat flux q_w are given by:

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{15}$$

with μ being the dynamic viscosity. Using the non-dimensional variables (6), we obtain:

$$\frac{1}{2} C_f \mathrm{Re}_x^{1/2} = f''(0), \quad Nu_x / \mathrm{Re}_x^{1/2} = 1/\theta(0), \tag{16}$$

where $Re_x = u_w x/\nu$ is the local Reynolds number.

Results and Discussion

The system of (7)-(9) was solved numerically using the Keller-box method described by Cebeci and Bradshaw (1988) for some values of parameters. Comparison with previously published data available in the literature as well as the series solution given by (13) for a particular case, as presented in Table 1 shows a good agreement.

As can be seen from Figure 2, when $\varepsilon = 1$, the velocity profiles for different values of M coincide, which means for this case the flow field is not influenced by the magnetic field. The numerical results are in agreement with the exact solution given by (10), which produces $f'(\eta) = 1$ and $f''(\eta) = 0$ for any values of η. The zero skin friction for this case ($\varepsilon = 1$) is not surprising since the surface and the fluid move with the same velocity. When $\varepsilon > 1$, the flow has a boundary layer structure, and the thickness of the boundary layer decreases with M. On the other hand, when $\varepsilon < 1$, the flow has an inverted boundary layer structure, which results from the fact that when $b/a < 1$, the stretching velocity ax of the surface exceeds the velocity bx of the external stream. For this case too, the thickness of the boundary layer decreases with M, which implies increasing manner of the magnitude of the velocity gradient at the surface. Thus, the magnitude of the skin friction coefficient $f''(0)$ increases with increasing M for both cases $\varepsilon > 1$ and $\varepsilon < 1$. The solution when $\varepsilon = 0$ is given by (11), which implies $f''(0) = -\sqrt{1+M}$.

Examining Table 1, for $n = -1$, the surface temperature is infinite, and in fact independent upon ε, Pr and M. This observation can also be obtained by integrating (8) with respect to η and applying the appropriate boundary conditions (9), which gives:

$$\int_0^\infty f'\theta \, d\eta = \frac{1}{(n+1)Pr}. \tag{17}$$

In fact, $n = -1$ is only possible for the prescribed surface temperature case, and it represents stretching surface subject to an adiabatic situation (Ishak et al. 2007, 2008d).

TABLE 1. Values of surface temperature $\theta(0)$

ε	n	Pr	M	Elbashbeshy (1998)	Liu (2005)	Present results	
						Numerical	Eq. (13)
0	0	0.72	0	2.13767		2.1591531	2.159153068
		1		1.71792		1.7182818	1.718281828
		10		0.43341		0.4332748	0.433274823
	1	0.72		1.2253		1.2366575	1.236657472
		1		1.0		1.0	1.0
		6.7		0.2688	0.333303	0.3333031	0.333303061
		10				0.2687685	0.268768515
			0.5		0.339715	0.3397152	0.339715220
			1		0.345377	0.3453772	0.345377171
			5		0.380930	0.3809302	0.380930205
0.1	-1					∞	
	0					1.8155361	
	1					1.0516189	
2	-1					∞	
	0					1.0142316	
	1					0.6673570	

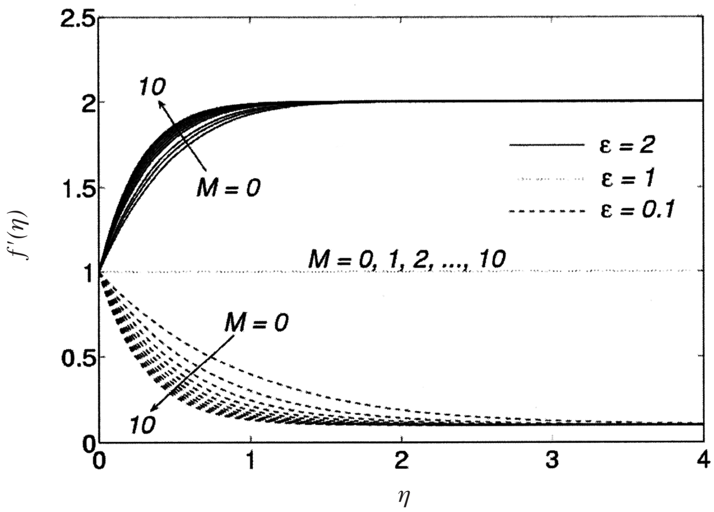

FIGURE 2. Velocity profiles $f'(\eta)$ for different values of M and ε

Figures 3 to 6 show the temperature profiles for selected values of parameters. The temperature profiles are found to subside monotonously to zero as η increases. These curves represent the physically realistic case. As can be seen from Figures 3 to 5, the wall temperature $\theta(0)$ decreases with increasing n, ε and Pr. Thus, the local Nusselt number $Nu_x/\mathrm{Re}_x^{1/2}$, which represents the heat transfer rate at the surface increases when n, ε or Pr increases. Moreover, Figures 3 to 5 show that at a given point in the thermal boundary layer, the temperature decreases with increasing n, ε or Pr, due to decreasing manner of the thermal boundary layer thickness with increasing these parameters. Different characteristics are observed in Figure 6, where the wall temperature increases as M increases for $\varepsilon < 1$, but it decreases with M for $\varepsilon > 1$.

CONCLUSION

We have theoretically investigated the effects of magnetic parameter M, velocity ratio parameter ε, heat flux index n, and Prandtl number Pr on the fluid flow and heat transfer characteristics of the MHD stagnation point flow towards a stretching sheet immersed in a viscous fluid. The numerical results obtained agreed very well with previously published data as well as the series solution for a particular case of the present study. It is found that the magnitude of the skin friction coefficient $|f''(0)|$ increases with M when $\varepsilon \neq 1$, and zero when $\varepsilon = 1$. Further, the heat transfer rate at the surface $1/\theta(0)$ increases with n, ε and Pr, while different behaviors are observed for variation with M, i.e. $1/\theta(0)$ increases with M when $\varepsilon > 1$ and it decreases when $\varepsilon < 1$.

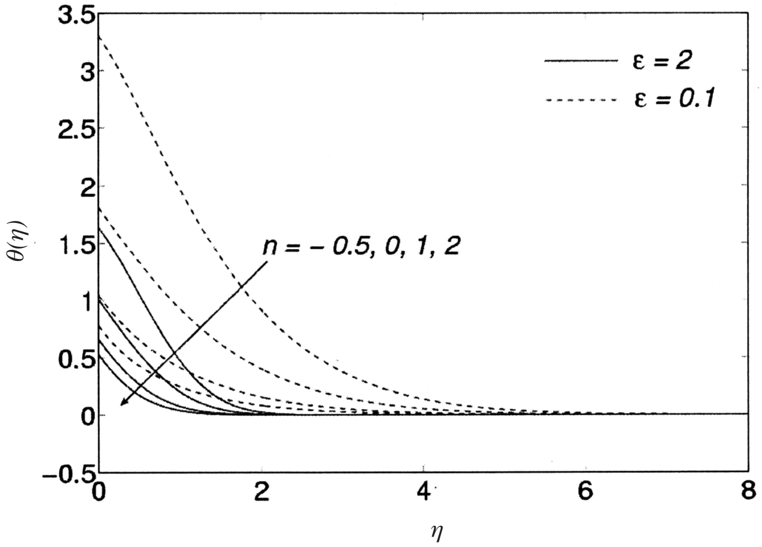

FIGURE 3. Temperature profiles $\theta(\eta)$ for different values of n and ε when Pr = 1 and $M = 1$

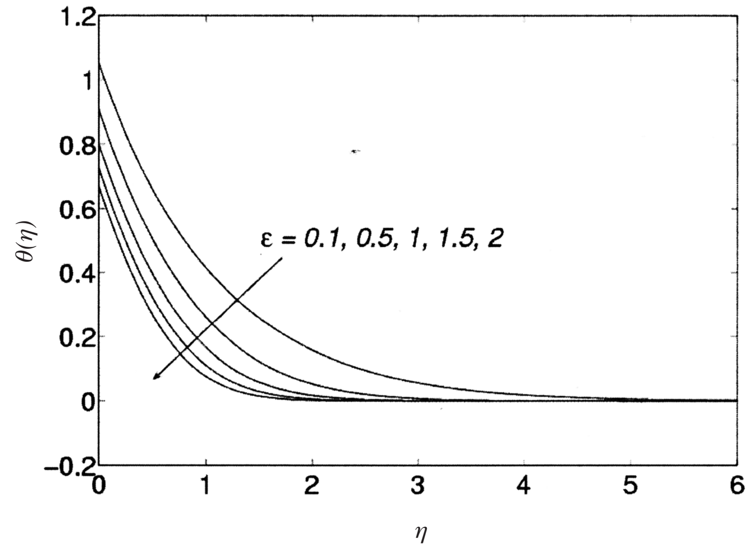

FIGURE 4. Temperature profiles $\theta(\eta)$ for different values of ε when Pr = 1, $M = 1$ and $n = 1$

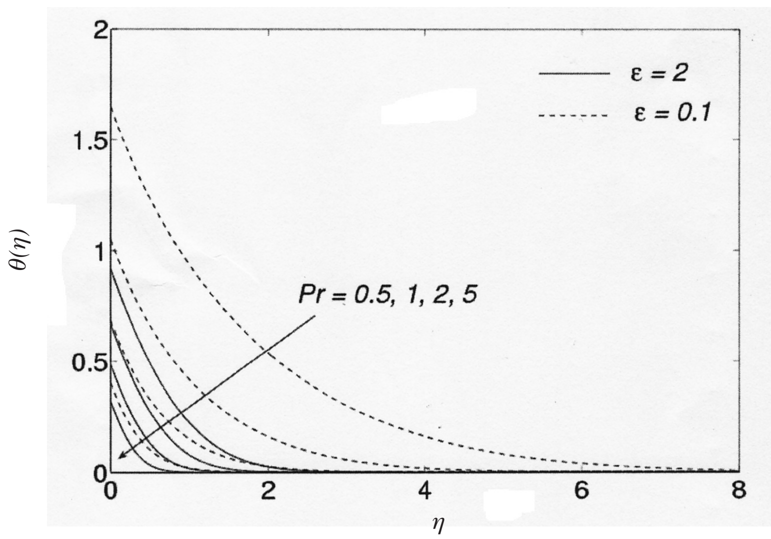

FIGURE 5. Temperature profiles $\theta(\eta)$ for different values of Pr and ε when $M = 1$ and $n = 1$

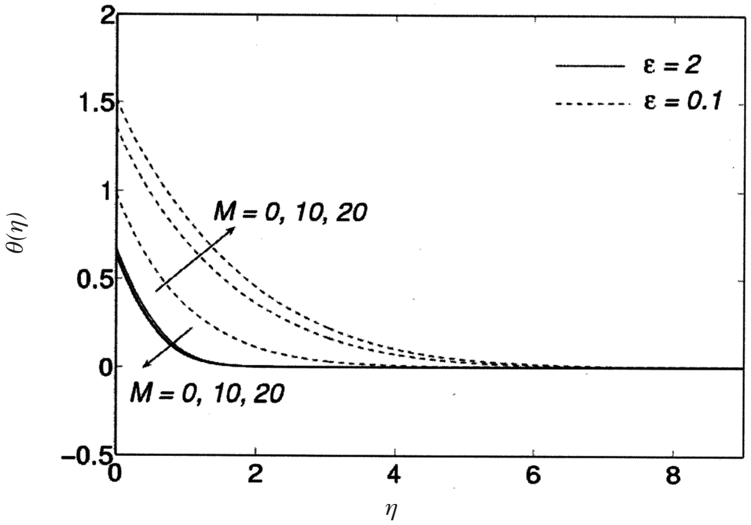

FIGURE 6. Temperature profiles $\theta(\eta)$ for different values of M and ε when Pr = 1 and n = 1

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